

The Pythagorean Theorem and Physics

The Pythagorean Theorem defines separation relations that must prevail if the universe is to exist. Some properties of Space resulting therefrom are explored.

Donald G. Burkhard
donalddbukhard@aol.com

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