# 20

# MIXED MULTIVECTORS

## 20.1 Mixed Multivector Interactions: Preliminary

For the interaction of multivectors of different kinds go back to Eqs. (20.4), (20.5). In this case, label one multivector with a prime, for example $q'_t$. The source multivector $q_s$ is unprimed; once again, roles are interchangeable. The combination $\left(q'_t q_s + q_s q'_t\right)/2$ is no longer a scalar. The structure depends on the particular multivectors $q'_t$ and $q_s$. The multivectors resulting from various $q'_t$ and $q_s$ are tabulated in Table A. For example, if $q'_t$ is electric charge (electromagnetism) and $q_s$ is a 4-momentum vector (gravity), then the effective charge is $\left(q'_t P_s + P_s q'_t\right)/2$, a vector quantity.

Magnetic field produced by $q_s \mathbf{V}_s$

$$\mathbf{B}'_s = k\mu_s^{em}\left(q_s \mathbf{V}_s \wedge \mathbf{r}_{st}\right) = k\mu_s^{em} q_s \mathbf{B}_s \tag{20.1}$$

Magnetic field produced by $P_t \mathbf{V}_t$

$$\mathbf{B}'_t = k\mu_t^{g}\left(P_t \mathbf{V}_t \times \mathbf{r}_{ts}\right) = k\mu_t^{g} P_t \mathbf{B}_t \tag{20.2}$$

Force of $q_s \mathbf{V}_s$ on $P_t \mathbf{V}_t$ is

$$\mathbf{F}_{ts} = k\mu_s^{em} q_s \mathbf{B}_s \bullet P_t \mathbf{V}_t, \qquad \mathbf{B}_s = \mathbf{V}_s \times \mathbf{r}_{st} \tag{20.3}$$

Force of $P_t \mathbf{V}_t$ on $q_s \mathbf{V}_s$ is

$$\mathbf{F}_{st} = k\mu_t^{g} P_t \mathbf{B}_t \bullet q_s \mathbf{V}_s \qquad \mathbf{B}_t = \mathbf{V}_t \times \mathbf{r}_{ts} \tag{20.4}$$

The result of placing the two completely different coefficients, $\mu_s^{em}$ in (20.3) and $\mu_t^g$ in (20.4) makes

$$\mathbf{F}_{ts} \neq \mathbf{F}_{st}$$

by virtue of these numerical factors alone. We know that $\mathbf{F}_{ts} \neq \mathbf{F}_{st}$ because of the structure of the interaction, but we know how to make $\mathbf{F}_{ts} = \mathbf{F}_{st}$ up to terms in $v^2/c^2$ for both electromagnetism and for gravity separately by including relativistic corrections to the fields and also including indirect forces. We now conjecture that the proper proportionality coefficient to insert in (20.3) and (20.4) is

$$\mu_{em}^g = (\mu_s^{em}\mu_t^g)^{1/2} \tag{20.5}$$

and re-write Eqs. (20.3), (20.4) as

$$\mathbf{F}_{ts} = k\mu_{em}^g q_s \mathbf{B}_s \bullet P_t \mathbf{V}_t \tag{20.6}$$

$$\mathbf{F}_{st} = k\mu_{em}^g P_t \mathbf{B}_t \bullet q_s \mathbf{V}_s \tag{20.7}$$

The labeling $\mu_{em}^g$ means that the proportionality constant applies when electromagnetic source charge (an electric charge) interacts with gravitational source charge (a momentum 4-vector). We assume that the relationship

$$\mu_{0,em}^g \varepsilon_{0,em}^g = \frac{1}{c^2}, \qquad \text{so} \qquad \varepsilon_{0,em}^g = \frac{1}{\mu_{em}^g c^2} \tag{20.8}$$

applies.

Equations (20.6) and (20.7) are now formally the same as equations that give the force between "like" multivectors. We assume the equivalent of the electric field is

$$\mathbf{E}_s = q_s/4\pi\varepsilon_{0,em}^g \tag{20.9}$$

Correspondingly, for the "interaction" between unlike multivectors, we recognize both direct and indirect "forces." For now we use the symbol $\mathbf{F}_{ts}$, although, in general, the result after contraction may be any of the multivectors: a vector, bivector, trivector, scalar or pseudo scalar; that is, not necessarily a force. It may be a torque, for example, which is one interpretation of a bivector. In the case of electromagnetism and gravity the interaction after contraction is a vector. Note that

$$(q_s P_t + P_t q_s)/2 = \text{ a vector}, \quad \text{and} \quad (q_s P_t - P_t q_s) = 0.$$

Thus the force of $q_s$ on $P_t$ is a vector. The second term is zero. Note that the second term is not zero for gravity-gravity interaction. It is always zero for scalar-scalar interaction (electromagnetism).

We summarize the above statements by writing the sum of (20.3) and (20.4) for the direct force before contraction.

$$\mathbf{F}_{ts}^{\mathrm{dir}} = \mu_{em}^{g} k \left[ (\mathbf{B}_s \bullet \mathbf{V}_t) (P_s q_t + q_t P_s)/2 + (\mathbf{B}_s \wedge \mathbf{V}_t) (P_s q_t - q_t P_s)/2 \right] \tag{20.10}$$

By Eqs. (20.3-20.5) the indirect "force" experienced by $q_t$ (electron) by a gravity source is

$$\begin{aligned} \mathbf{F}_{t}^{\mathrm{indir}} &= \mathbf{F}_{ts}^{\mathrm{indir}}(+) + \mathbf{F}_{ts}^{\mathrm{indir}}(-) \\ &= \mathbf{F}_s \bullet (\mathbf{V}_t + \mathbf{e}_5 \mathbf{U}_t) (P_s q_t + q_t P_s)/2 \\ &\quad + \mathbf{F}_s \wedge (\mathbf{V}_t + \mathbf{e}_5 \mathbf{U}_t) (P_s q_t - q_t P_s)/2 \end{aligned} \tag{20.11}$$

The expressions to be used for $\mathbf{V}_s$ and $\mathbf{U}_s$ when calculating indirect forces are

$$\mathbf{V}_t = \left( \mathbf{e}_0 \mathrm{div} \frac{\mathbf{E}_t}{c} - \frac{1}{c^2} \frac{\partial \mathbf{E}_t}{\partial t} + \mathrm{curl}\, \mathbf{B}_t \right) \tag{20.12}$$

$$\mathbf{U}_t = \left( -\mathbf{e}_0 \mathrm{div}\, \mathbf{B}_t + \frac{\partial \mathbf{B}_t}{\partial t} + \mathrm{curl}\, \mathbf{E}_t \right) \tag{20.13}$$

Also when evaluating the indirect force, $\mathbf{B}_s$ in Eq. (20.10) is replaced by $\mathbf{F}_s$ where

$$\mathbf{F}_s = \frac{E_{sx}}{c} \mathbf{e}_0 \mathbf{e}_1 + \frac{E_{sy}}{c} \mathbf{e}_0 \mathbf{e}_2 + \frac{E_{sz}}{c} \mathbf{e}_0 \mathbf{e}_3 + B_{sz} \mathbf{e}_1 \mathbf{e}_2 + B_{sy} \mathbf{e}_3 \mathbf{e}_1 + B_{sx} \mathbf{e}_2 \mathbf{e}_3 \tag{20.14}$$

If source expressions are used for $\mathbf{E}_t$ and $\mathbf{B}_t$, as they will be later, then they are given by

$$\mathbf{E}_t = \frac{1}{4\pi \varepsilon_{0,em}^{g}}, \qquad \mathbf{B}_t = \frac{\mu_{0,em}^{g} (\mathbf{v}_t \times \mathbf{r})}{4\pi r^3} = \frac{\mathbf{v}_t \times \mathbf{r}}{4\pi \varepsilon_{0,em}^{g} c^2 r^3} \tag{20.15}$$

In Eq. (20.12) the only change in form from that of two like multivector interaction is the replacement of the usual $\varepsilon_0$ and $\mu_0$ by the above versions as defined by Eq. (20.5). One may make (20.12) and (20.13) more explicit by adding labels to $\mathbf{E}_t$ and $\mathbf{B}_t$, perhaps $\mathbf{E}_{t,em}^{g}$ and $\mathbf{B}_{t,em}^{g}$. Rather than introduce more notational clutter we simply state that the values of $\mathbf{E}_t$ and $\mathbf{B}_t$ in (20.12) and (20.13) are given by (20.14) when source values are required. When referring to the free field, that is, the field without reference to its sources, it is understood that $\mathbf{E}$ and $\mathbf{B}$ are generated by gravity (4 momentum) and electromagnetism (electric charge) as sources.