17

LIGHT IN A SCHWARZCHILD METRIC. PLANCK VALUES.

17.1 Velocity of Light in Various Directions in a Schwarzschild metric

ds in the Schwarzschild Metric is:

$$ds = \mathbf{e}_0 \left(1 - \frac{2GM}{rc^2}\right)^{1/2} cdt + \mathbf{e}_r \left(1 - \frac{2GM}{rc^2}\right)^{-1/2} dr + \mathbf{e}_\theta r d\theta + \mathbf{e}_\varphi r \sin\theta d\varphi$$

Let $\alpha = GM/c^2$

$$ds = \mathbf{e}_0 \left(1 - 2\alpha/r\right)^{1/2} + \mathbf{e}_r \left(1 - 2\alpha/r\right)^{-1/2} dr + \mathbf{e}_\theta r d\theta + \mathbf{e}_\varphi r \sin\theta d\varphi$$

ds is zero if the normal light velocity is c. Then the square of the Schwarzschild metric is:

$$0 = ds^2 = -\left(1 - 2\alpha/r\right)\left(c^2 dt^2\right) + \left(1 - 2\alpha/r\right)^{-1}(dr)^2 + r^2 (d\theta)^2 + r^2 \sin^2\theta\, (d\varphi)^2$$

Taking $dr = 0$ and $d\varphi = 0$,

$$-\left(1 - 2\alpha/r\right)\left(c^2 dt^2\right) + r^2 (d\theta)^2 = 0$$

$$r\frac{d\theta}{dt} = c\sqrt{1 - 2\alpha/r}$$

Taking $d\theta = 0$ and $d\varphi = 0$,

$$-(1-2\alpha/r)\left(c^2dt^2\right)+(1-2\alpha/r)^{-1}(dr)^2=0$$

$$\frac{dr}{dt}=\frac{\sqrt{(1-2\alpha/r)}}{\sqrt{(1-2\alpha/r)^{-1}}}c=(1-2\alpha/r)\,c$$

Taking $\sin\theta = 1$, $d\theta = 0$, $dr = 0$

$$-(1-2\alpha/r)\left(c^2dt^2\right)+r^2(d\varphi)^2=0$$

$$r\frac{d\varphi}{dt}=\left(\sqrt{(1-2\alpha/r)/c}\right)$$

$$c_\varphi = c\sqrt{1-2\alpha/r}$$

$$\begin{aligned}
1-v^2/c_\varphi^2 &= 1-\frac{v^2}{c^2(1-2\alpha/r)}\cong 1-\frac{v^2}{c^2}(1-2a/r)\\
1-v^2/c_\varphi^2 &\cong 1-\frac{v^2}{c^2}-\frac{v^2}{c^2}\frac{2v^2}{c^2}\\
1-v^2/c_\varphi^2 &\cong 1-\frac{v^2}{c^2}-2\frac{v^4}{c^4}
\end{aligned}$$

To be consistent, corrections of higher order than v^2/c^2 must also be included in flat space if applied at all. The above higher order corrections to v^2/c^2 may be neglected here since we are only interested in an accuracy to v^2/c^2. The reference for the above is: Mizushima, M. 2000. General relativistic optics and related gravitational effects. *Hadronic Journal* 23(3): 273-278.

For applications of the Einstein metric see other papers by Mizushima in the Hadronic Journal.

17.2 Planck Values for M, L, T and Other Physical Quantities

Dimensions of G, h, and c

$$\begin{aligned}
\frac{GMm}{R^2} &= \frac{mv^2}{R}\\
G &= \frac{v^2R}{M}=M^{-1}L^3T^{-2}
\end{aligned}$$

$h\nu$ =energy

$$h = \frac{MV^2}{\nu} = \frac{ML^2T}{T^2} = ML^2T^{-1}$$
$$c = LT^{-1}$$

Equate the product of the powers of the quantities G, h, c to the fundamental values of M, L, T called the Planck values designated M_{pe}, L_{pe}, T_{pe}. In the following, we omit the subscripts.

G^x	h^y	c^z
$(M^{-1}L^3T^{-2})^x$	$(ML^2T^{-1})^y$	$(LT^{-1})^z$
M^{-x+y}	$L^{3x+2y+z}$	$T^{-2x-y-z}$

1. $M_{p\ell}$ $-x+y=1$, $3x+2y+z=0$, $-2x-y-z=0$ from which $x=-1/2$, $y=1/2$, $z=1/2$ $M^1L^0T^0$

2. $L_{p\ell}$ $-x+y=0$, $3x+2y+z=1$, $-2x-y-z=0$ from which $x=1/2$, $y=1/2$, $z=3/2$ $M^0L^1T^0$

3. $T_{p\ell}$ $-x+y=0$, $3x+2y+z=0$, $-2x-y-z=1$ from which $x=1/2$, $y=1/2$, $z=-5/2$ $M^0L^0T^{-1}$

4. Planck Energy, Mv^2
   $E_{p\ell}$ $-x+y=1$, $3x+2y+z=2$, $-2x-y-z=-2$ from which $x=-1/2$, $y=1/2$, $z=5/2$ ML^2T^{-2}

Thus

$$\begin{aligned}
1.\ G^{-1/2}\hbar^{1/2}c^{1/2} &= M^1L^0T^0\\
2.\ G^{1/2}\hbar^{1/2}c^{-3/2} &= M^0L^1T^0\\
3.\ G^{1/2}\hbar^{1/2}c^{-5/2} &= M^0L^0T^1\\
4.\ G^{-1/2}\hbar^{1/2}c^{5/2} &= M^1L^2T^{-2}
\end{aligned}$$

Numeric Values

$$
\begin{aligned}
\hbar &= \frac{h}{2\pi} = 1.05 \times 10^{-34} \ \text{joules} \cdot \ \text{sec} \\
G &= 6.67 \times 10^{11} \ \frac{\text{Newton-(meter)}^2}{(\text{kg})^2} \\
c &= 3 \times 10^8 \ \text{meters/sec} \\
L_{p\ell} &= \left(\frac{G\hbar}{c^3}\right)^{1/2} = \left(\frac{(6.67 \times 10^{11})(1.05 \times 10^{-34})}{(3 \times 10^8)^3}\right)^{1/2} = 1.62 \times 10^{-24} \ \text{meters} \\
T_{p\ell} &= \left(\frac{G\hbar}{c^5}\right)^{1/2} = \left(\frac{(6.67 \times 10^{11})(1.05 \times 10^{-34})}{(3 \times 10^8)^5}\right)^{1/2} = 5.4 \times 10^{-33} \ \text{sec} \\
M_{p\ell} &= \left(\frac{\hbar c}{G}\right)^{1/2} = \left(\frac{(1.05 \times 10^{-34})(3 \times 10^8)}{(6.67 \times 10^{11})}\right)^{1/2} = 2.17 \times 10^{-19} \ \text{kg} \\
E_p &= \left(\frac{\hbar c}{G}\right)^{1/2} \left(\frac{G\hbar}{c^3}\right) \left(\frac{c^5}{G\hbar}\right) = G^{-1/2}\hbar^{1/2}c^{5/2} \\
&= \left(\frac{(1.05 \times 10^{-34})(3 \times 10^8)^5}{(6.67 \times 10^{11})}\right)^{1/2} = 1.98 \times 10^{-2} \ \text{joule} = 1.025 \times 10^8 \ \text{GeV}
\end{aligned}
$$

$$
\begin{aligned}
\text{Force:} \qquad P\ell &= MA = \frac{L}{T^2} = \left(\frac{\hbar c}{G}\right)^{1/2} \left(\frac{G\hbar}{c^3}\right)^{1/2} \left[\left(\frac{c^5}{G\hbar}\right)^{1/2}\right]^2 \\
&= \frac{c^4}{G} = \frac{(3 \times 10^8)^4}{6.67 \times 10^{11}} = 1.21 \times 10^{22} \ \text{Newton}
\end{aligned}
$$

$$
\begin{aligned}
\text{Torque:} \quad P\ell &= F\ell = \frac{c^4}{G} \times \left(\frac{G\hbar}{c^3}\right)^{1/2} = \frac{c^{5/2}\hbar^{1/2}}{G^{1/2}} \\
&= \frac{(3 \times 10^8)^{5/2}(1.05 \times 10^{-34})^{1/2}}{(6.67 \times 10^{11})^{1/2}} = 1.96 \times 10^{-2} \ \text{joule}
\end{aligned}
$$

or simply, the above values substituted in

$$
F \times L = 1.21 \times 10^{22} \times 1.62 \times 10^{-24} = 1.96 \times 10^{-2} \ \text{Newton meters}
$$

Angular Momentum: $P\ell = \hbar = 1.05 \times 10^{-34} \text{joule} \cdot \text{sec}$

Angular Momentum Density:
$$P\ell = \frac{\hbar}{L_{p\ell}^3} = \hbar \left(\frac{c^3}{G\hbar}\right)^{3/2}$$
$$= \frac{1.05 \times 10^{-34}}{(1.62 \times 10^{-24})^3} = 3.3 \times 10^{38} \ \frac{\text{joule} \cdot \text{sec}}{(\text{meter})^3}$$

Torque Density:
$$P\ell = \frac{c^{5/2}\hbar^{1/2}}{G^{1/2}} \left(\frac{c^3}{G\hbar}\right)^{3/2}$$
$$= \frac{c^7}{G^2\hbar} = \frac{(3 \times 10^8)^7}{(6.67 \times 10^{11})^2 (1.05 \times 10^{-34})}$$
$$= 4.68 \times 10^{69} \ \frac{\text{joule}}{(\text{meter})^3}$$

or, as a check,

Torque Density:
$$P\ell = \frac{\text{Torque}}{(1.62 \times 10^{-24})^3}$$
$$= \frac{1.96 \times 10^{-2}}{(1.62 \times 10^{-24})^3}$$
$$= 4.68 \times 10^{69} \ \frac{\text{Newton} \cdot \text{meter}}{(\text{meter})^3}$$