11

EVALUATION OF THE TERM $\epsilon_o\left(\mathbf{E}_t \times \mathbf{B}_t\right)$ IN THE GENERAL EQUATION FOR LINEAR MOMENTUM

For the linear momentum density imparted to the field by two particles with velocities $\dot{\mathbf{v}}_s$ and $\mathbf{v}_t$, for electromagnetism we had

$$\begin{aligned} g_\ell &= \epsilon_0(\mathbf{E}_t + \mathbf{B}_t) \times (\mathbf{B}_t + \mathbf{B}_s) \\ &= \epsilon_0\left[(\mathbf{E}_t \times \mathbf{B}_t) + (\mathbf{E}_t \times \mathbf{B}_s) + (\mathbf{E}_s \times \mathbf{B}_t) + (\mathbf{E}_s \times \mathbf{B}_s)\right] \end{aligned} \quad (11.1)$$

The contribution of the two middle terms to the force on each other has been evaluated. The gravitation field, as has been shown, has the same structure as the electromagnetic field. Therefore, we have used the same symbols, $\mathbf{E}$ and $\mathbf{B}$, to describe both fields. The gravitational "charge" is $m_s\mathbf{V}_s$ and $m_t\mathbf{V}_t$ (not simply m_s and m_t) where $\mathbf{V}_s$ and $\mathbf{V}_t$ are 4-dimensional space-time 4-velocities.

11.1 Reactive Force on a Moving Mass

For evaluation of $\epsilon_o\left(\mathbf{E}_t \times \mathbf{B}_t\right)$, see Figures 11.1 and 11.2 which describe the radiation of a single charge in Feynman et al. 1977 and Page 1952. In the present case, the Figures in Feynman also describes the end terms $\epsilon_o\left(\mathbf{E}_s \times \mathbf{B}_s\right)$ and $\epsilon_o\left(\mathbf{E}_t \times \mathbf{B}_t\right)$, Eq. 11.1.

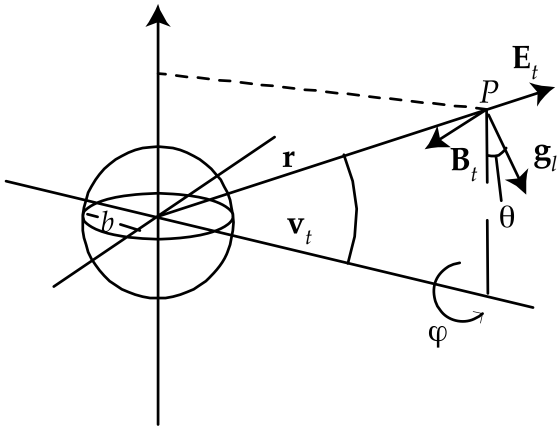

Fig. 11.1

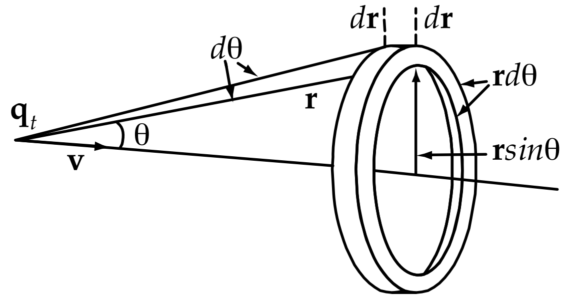

Fig. 11.2

Consider a spherical source radius b. For an attractive source $\mathbf{E}$ and $\mathbf{B}$ are reversed but $\mathbf{g}_\ell$ is not.

$\mathbf{g}_\ell = \varepsilon_0\left(\mathbf{E}_t \times \mathbf{B}_t\right)$ is the contribution momentum density in the field where $\mathbf{E}$ and $\mathbf{B}$ have the values appearing in Eq. (11.1). The integral of $\mathbf{g}_\ell$ over all space, that is,

$$\mathbf{G}_\ell = \int \mathbf{g}_\ell d\tau = \int \varepsilon_0\left(\mathbf{E}_t \times \mathbf{B}_t\right) d\tau \tag{11.2}$$

gives the total linear momentum in the field associated with the values of $\mathbf{E}$ and $\mathbf{B}$ existing throughout the field. The source in this case is what we have called the test particle.

The magnitude of B_t is

$$B_t = \frac{v_t}{c^2} E_t$$

$\mathbf{g}_\ell$ is directed obliquely toward the line of motion and has the magnitude g_ℓ along v_t where

$$g_\ell = \varepsilon_0 q_t^2 E_t B_t = \varepsilon_0 q_t^2 \frac{v_t}{c^2} E_t^2 \sin\theta \tag{11.3}$$

The fields are symmetric about the line of motion, so when integrated over all space the transverse components will sum to zero, giving a resultant momentum parallel to $\mathbf{v}_t$. The component of $\mathbf{g}$ in this direction is $g\sin\theta$ which must be integrated over all space. Take volume element as a ring with its plane perpendicular to $\mathbf{v}_t$ as shown in Fig.11.2.

The volume element is

$$d\tau = 2\pi r^2 \sin\theta d\theta dr$$

The total momentum $\mathbf{G}$ in the $\mathbf{v}$ direction for gravity is

$$\mathbf{G} = \frac{\varepsilon_0 \mathbf{v}_t}{c^2} q_t^2 \int E^2 \sin\theta 2\pi r^2 \sin\theta d\theta dr$$

The integral in θ is

$$\begin{aligned}\int_0^\pi \sin^3\theta d\theta &= -\int_0^\pi \left(1-\cos^2\theta\right) d\left(\cos\theta\right) = \left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_0^\pi \\ &= 1+1-\frac{1}{3}-\frac{1}{3} = \frac{4}{3}\end{aligned}$$

$$\begin{aligned}\mathbf{G}_\ell &= 2\left(\frac{4\pi}{3}\right)\frac{\varepsilon_0 \mathbf{v}_t}{c^2} q_t^2 \int_\infty^b E^2 r^2 dr \\ E^2 &= \frac{q_t^2}{\left(4\pi\varepsilon_0\right)^2 r^4}, \qquad \int_\infty^b E^2 r^2 dr = \frac{q_t^2}{\left(4\pi\varepsilon_0\right)^2}\left.\frac{1}{r}\right]_\infty^b = \frac{q_t^2}{\left(4\pi\varepsilon_0\right)^2}\frac{1}{b} \\ \mathbf{G}_\ell &= \frac{2\left(4\pi\right)\varepsilon_0 \mathbf{v}_t}{3c^2}\frac{q_t^2}{\left(4\pi\varepsilon_0\right)^2 b} = \frac{2}{3}\frac{q_t^2}{\left(4\pi\varepsilon_0\right)}\frac{\mathbf{v}_t}{bc^2} \text{ (mks units)} \quad = \frac{2q_t^2\mathbf{v}_t}{3bc^2} \text{ (cgs units)}\end{aligned} \quad (11.4)$$

In Eq. (11.4), we will equate the coefficient of $\mathbf{v}_t$ to the measured mass of the particle so that

$$\mathbf{G}_\ell = m_t \mathbf{v}_t \quad (11.5)$$

To obtain the relativistic velocity, substitute $\mathbf{v}_t/\sqrt{1-v^2/c^2}$ for $\mathbf{v}_t$. Then

$$\mathbf{G}_\ell = \frac{q_t^2}{4\pi\varepsilon_0 bc^2}\frac{\mathbf{v}_t}{\left(1-v^2/c^2\right)^{1/2}} = \frac{m_t\mathbf{v}_t}{\left(1-v^2/c^2\right)^{1/2}} \quad (11.6)$$

All $\mathbf{v}_s'$ are $\mathbf{v}_t$. $\mathbf{a}_s'$ are $\mathbf{a}_t$.

Therefore, if $\mathbf{a}$ is the acceleration of the mass m_t, the force on the force field, is

$$\frac{d\mathbf{G}}{dt} = m_t\left(\frac{\mathbf{a}}{\left(1-v^2/c^2\right)^{1/2}} + \frac{\mathbf{v}\left(\mathbf{v}\cdot\mathbf{a}\right)}{c^2\left(1-v^2/c^2\right)^{3/2}}\right) \quad (11.7)$$

.

Forming the derivatives of a space-time 4-velocity as in Eq. (11.7) is equivalent to introducing $q_t = m_t v_t$ for gravitational charge. See Section 4.4 for 4-acceleration.

The component of the force parallel to the velocity is

$$\mathbf{f}_\parallel = \frac{d\mathbf{G}_\parallel}{dt} = \frac{m\mathbf{a}_\parallel}{\left(1-\beta^2\right)^{3/2}} \qquad m = \frac{2e^2}{bc^2}$$

The component of the force perpendicular to the velocity is

$$\begin{aligned}\mathbf{f}_\perp &= \frac{d\mathbf{G}_\perp}{dt} = \frac{m\mathbf{a}_\perp}{\left(1-\beta^2\right)^{1/2}} = \frac{mv^2}{\left(1-\beta^2\right)^{1/2} r} \\ &\cong m\frac{v^2}{r}\left(1+\frac{v^2}{2c^2}\right) \end{aligned} \tag{11.8}$$

$\mathbf{a}_\parallel$ and $\mathbf{a}_\perp$ are the components of $\mathbf{a}$ parallel to and perpendicular, respectively, to the velocity.

For $\mathbf{a}\perp\mathbf{v}$ and $\mathbf{a}_\perp = \frac{v^2}{R}\hat{\mathbf{r}}$, motion in a circle,

$$\frac{d\mathbf{G}_\ell}{dt} = m\left(\frac{v^2}{R}\right)\hat{\mathbf{r}} \tag{11.9}$$

To quote Page (1952, 551), referring to electromagnetism, "[Newton's third law] requires that the electron's field exert an equal and opposite force on the electron. So the components of the force exerted on the electron are proportional to $\mathbf{f}_\parallel$ and $\mathbf{f}_\perp$ and opposite in direction. Thus we have a purely electromagnetic explanation of the reaction of any charged particle. Since matter is composed of charged particles, we have deduced Newton's second law of motion from electromagnetic theory."

At this stage the idea of a mass change with velocity is often introduced so that a so-called longitudinal mass $m_\ell = m_0/\left(1-v^2/c^2\right)^{3/2}$ and a transverse mass, $m_t = m_0\sqrt{1-v^2/c^2}$ are defined. m_0 is the rest mass. Instead, we regard mass as an invariant and treat the time rate of change of velocity as having two components. Then invariant mass is:

$$m = 2\left(\frac{q_s^2}{4\pi\varepsilon_0}\right)\frac{1}{bc^2} \tag{11.10}$$

For a charge (mass) moving in a circle, with relativistic velocity

$$\begin{aligned}\mathbf{f}_\perp &= \frac{2q_s^2\mathbf{a}_\perp}{bc^2 4\pi\varepsilon_0\sqrt{1-\beta^2}} = \frac{2q_s^2 v^2\hat{\mathbf{r}}}{bc^2 4\pi\varepsilon_0 R\sqrt{1-\beta^2}} = \frac{mv^2\hat{\mathbf{r}}_\perp}{R\sqrt{1-\beta^2}} \\ &\cong m\frac{v^2}{R}\left(1+\frac{v^2}{2c^2}\right)\hat{\mathbf{r}} \end{aligned} \tag{11.11}$$

Since

$$\frac{Mv^2}{R} = \frac{mMG}{R^2} \tag{11.12}$$

We obtain

$$\mathbf{f}_\perp = \frac{mMG}{R^2}\left(1+\frac{v^2}{2c^2}\right)\hat{\mathbf{r}} \tag{11.13}$$

11.2 Effective Value of G Resulting from the Circular Motion of a Single Charge (Mass) Around a Large Mass

The above yields a value for G of

$$G\left(1+\frac{v^2}{2c^2}\right) \tag{11.14}$$

.

When torque density is included, the net effective G is

$$G\left(1+\frac{v^2}{c^2}\right) \tag{11.15}$$

This is contribution number 4 in the Summary of Sources, Section 2.4, in Chapter 2 of v^2/c^2 that collectively account for the advance of the perihelion of Mercury.